

Name: _____ Date: _____ Class: _____

ESSENTIAL QUESTION

How do we quantify uncertainty? How can counting techniques help us calculate the likelihood of complex events?

Lesson Overview

Probability measures how likely an event is to occur, expressed as a number between 0 (impossible) and 1 (certain). The sample space S is the set of all possible outcomes; an event E is any subset of S .

$$P(E) = |E|/|S|$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = P(A \cap B)/P(B)$$

Classical Probability $P(E) = |E| / |S|$ (equally likely outcomes).

Complement Rule $P(E') = 1 - P(E)$.

Addition Rule $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. If mutually exclusive, $P(A \cap B) = 0$.

Independent Events $P(A \cap B) = P(A) \cdot P(B)$.

Conditional Probability $P(A|B) = P(A \cap B) / P(B)$.

Worked Examples**EXAMPLE 1**

Roll a fair six-sided die. Find $P(\text{even})$.

- Sample space: $S = \{1, 2, 3, 4, 5, 6\}$, $|S| = 6$. Event: $E = \{2, 4, 6\}$, $|E| = 3$.
- $P(E) = |E|/|S| = 3/6$

Answer: $P(\text{even}) = 1/2$

EXAMPLE 2

Draw one card from a standard 52-card deck. Find $P(\text{heart or face card})$.

- Hearts: $|A| = 13$. Face cards (J, Q, K each suit): $|B| = 12$. Both ($J\heartsuit, Q\heartsuit, K\heartsuit$): $|A \cap B| = 3$.
- $P(A \cup B) = 13/52 + 12/52 - 3/52 = 22/52 = 11/26$

Answer: $P(\text{heart or face card}) = 11/26 \approx 0.423$

EXAMPLE 3

Two fair coins are flipped. Find $P(\text{at least one head})$.

- Sample space: $S = \{HH, HT, TH, TT\}$, $|S| = 4$. Use the complement.
- $P(\text{no heads}) = P(TT) = 1/4$; $P(\text{at least one head}) = 1 - 1/4$

Answer: $P(\text{at least one head}) = 3/4$

EXAMPLE 4

A bag has 3 red, 5 blue, and 2 green marbles (10 total). Two marbles are drawn without replacement. Find $P(\text{both red})$.

- Total ways to choose 2 from 10: $C(10,2) = 45$. Ways to choose 2 red from 3: $C(3,2) = 3$.
- $P(\text{both red}) = C(3,2)/C(10,2) = 3/45$

Answer: $P(\text{both red}) = 1/15 \approx 0.067$

EXAMPLE 5

$P(A) = 0.4$, $P(B) = 0.3$, $P(A \cap B) = 0.1$. Find $P(A|B)$.

- $P(A|B) = P(A \cap B) / P(B) = 0.1 / 0.3$
- Interpretation: given that B has occurred, there is a 33.3% chance A also occurs.

Answer: $P(A|B) = 1/3 \approx 0.333$

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

- 1** Roll a fair six-sided die. Find $P(\text{prime number})$.

Hint: The prime numbers on a die are 2, 3, and 5. How many outcomes are in the sample space?

- 2** Draw one card from a 52-card deck. Find $P(\text{red or king})$.

Hint: Use the addition rule. Count red cards (26), kings (4), and red kings (2). Don't forget to subtract the overlap!

- 3** $P(A) = 0.5$, $P(B) = 0.4$, and A and B are mutually exclusive. Find $P(A \cup B)$.

Hint: Mutually exclusive means $P(A \cap B) = 0$. The addition rule simplifies to $P(A \cup B) = P(A) + P(B)$.

- 4** A jar has 4 red and 6 blue marbles (10 total). Two are drawn without replacement. Find $P(\text{one red and one blue})$.

Hint: Count favorable outcomes: $C(4,1) \cdot C(6,1)$. Divide by total ways to choose 2 from 10: $C(10,2) = 45$.

5 $P(A) = 0.6$, $P(B) = 0.5$, $P(A \cap B) = 0.3$. Find $P(A|B)$ and $P(B|A)$.

Hint: Use $P(A|B) = P(A \cap B)/P(B)$ and $P(B|A) = P(A \cap B)/P(A)$. Substitute carefully.

Key Vocabulary

Sample Space (S)	The set of all possible outcomes of a random experiment.
Event (E)	Any subset of the sample space; a collection of outcomes.
Complement (E')	All outcomes NOT in E; $P(E') = 1 - P(E)$.
Mutually Exclusive	Events that cannot occur simultaneously; $P(A \cap B) = 0$.
Independent Events	Events where the occurrence of one does not affect the other; $P(A \cap B) = P(A) \cdot P(B)$.
Conditional Probability	$P(A B) = P(A \cap B)/P(B)$; the probability of A given B has occurred.

Quick Check Quiz

Circle the letter of the correct answer.

1

A fair die is rolled. Which formula gives $P(A \cup B)$ when A and B are NOT mutually exclusive?

Multiple Choice

A $P(A) + P(B)$

B $P(A) + P(B) - P(A \cap B)$

C $P(A) \cdot P(B)$

D $P(A \cap B) / P(B)$

Common Mistakes to Avoid

✗ $P(A \cup B) = P(A) + P(B)$ always.

✓ Only when A and B are mutually exclusive. Otherwise subtract $P(A \cap B)$ to avoid double-counting.

✗ Confusing $P(A|B)$ with $P(B|A)$.

✓ These are generally different. $P(A|B) = P(A \cap B)/P(B)$ and $P(B|A) = P(A \cap B)/P(A)$.

✗ Assuming independent events are mutually exclusive.

✓ Independent means $P(A \cap B) = P(A) \cdot P(B) \neq 0$. Mutually exclusive means $P(A \cap B) = 0$. They are opposite concepts (for non-trivial events).

✗ Forgetting to reduce the denominator when drawing without replacement.

✓ After drawing one marble, there are $n-1$ items left. Use combinations: $C(n,k)$ for unordered draws.

Math Tips

- ★ Always check: does your probability answer fall between 0 and 1? If not, recheck your work.
- ★ The complement rule $P(E') = 1 - P(E)$ is often the fastest path when 'at least one' appears.
- ★ For independent events, $P(A \cap B) = P(A) \cdot P(B)$ — this is both the definition and a test for independence.
- ★ Draw a Venn diagram or tree diagram to visualize overlapping events before calculating.
- ★ When using combinations for probability, set up the fraction: (favorable combinations) / (total combinations).